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Mathematics paper 1 solutions 2024

Item 1

Two friends, Sarah and Moses started a poultry project to which they each contributed 4,000,000 and 6,000,000 respectively.

They agreed to share the profits in the ratio of their contributions and the project started with 2000 birds

After selling all the birds, they made a profit of one million five hundred thousand shillings. They reinvested the profit in the project and the number of birds increased to 2500. However Moses was not sure of the amount he reinvested

On 1st march 2024, they bought feeds and also balanced books of accounts. On that very day they had a meeting in which they agreed to buy feeds every after 7 days and balance their books of accounts every after 15 days.

Task

- How much profit did Moses re invest in the business
- What was the percentage increase in the number of birds
- On what date in future would they buy the feeds and also balance the books of account on the same day

Solution number 1

Summary per phrase of the question

Two friends, Sarah and Moses started a poultry project to which they each contributed 4,000,000 and 6,000,000 respectively

Sarah	4 million
Moses	6 million

They agreed to share the profits in the ratio of their contributions

$$= \frac{4M}{6M} = 4:6, \text{ Sarah: Moses}$$

After selling all the birds, they made a profit of one million five hundred thousand shillings. They reinvested the profit in the project.

particular	Capital invested	Profit / Amount reinvested
Sarah	4M	$\frac{4}{10} \times 1.5M = 600K$
Mosses	6M	$\frac{6}{10} \times 1.5M = 900K$ OR $1.5M - 600K = 900K$

a) Moses reinvested

$$= \frac{6}{(6+4)} \times 1.5M = \text{shs } 900,000$$

b) % increase = $\frac{\text{increase}}{\text{original}} \times 100 = \frac{2500-2000}{2000} \times 100 = 25\%$

c) This day is the LCM of 7 and 15

Method 1

Multiples of 7 are (7,14,21,28...105)

Multiples of 15 are (15, 30,45,60... 105)

LCM is the first multiple which appears in both sets = 105

They will buy feeds and balance books after 105 days concurrently

According to the 2024 calendar, 105 days after 1st march will happen on

2024

January							February							March							April						
Su	M	Tu	W	Th	F	Sa	Su	M	Tu	W	Th	F	Sa	Su	M	Tu	W	Th	F	Sa	Su	M	Tu	W	Th	F	Sa
1	2	3	4	5	6				1	2	3			3	4	5	6	7	8	9	1	2					
7	8	9	10	11	12	13	4	5	6	7	8	9	10	10	11	12	13	14	15	16	7	8	9	10	11	12	13
14	15	16	17	18	19	20	11	12	13	14	15	16	17	17	18	19	20	21	22	23	14	15	16	17	18	19	20
21	22	23	24	25	26	27	18	19	20	21	22	23	24	24	25	26	27	28	29	30	21	22	23	24	25	26	27
28	29	30	31				25	26	27	28	29			31							28	29	30				
May							June							July							August						
Su	M	Tu	W	Th	F	Sa	Su	M	Tu	W	Th	F	Sa	Su	M	Tu	W	Th	F	Sa	Su	M	Tu	W	Th	F	Sa
			1	2	3	4					1		1	2	3	4	5	6				1	2	3			
5	6	7	8	9	10	11	2	3	4	5	6	7	8	7	8	9	10	11	12	13	4	5	6	7	8	9	10
12	13	14	15	16	17	18	9	10	11	12	13	14	15	14	15	16	17	18	19	20	11	12	13	14	15	16	17
19	20	21	22	23	24	25	16	17	18	19	20	21	22	21	22	23	24	25	26	27	18	19	20	21	22	23	24
26	27	28	29	30	31		23	24	25	26	27	28	29	28	29	30	31				25	26	27	28	29	30	31
							30																				

March 01, 2024

plus

105 Days

June 14, 2024

Method 2 of multiplying prime factors

Days the 2 happen together	Buy feeds	Balance books
Buying feeds 7 times	7	15
Balancing books 15 times	1	15
	1	1

Which is 7×15 times = 105 days

ITEM 2

A tailor makes school uniforms for boys and girls. The tailor makes at least 80 sets of uniforms and not more than 100 sets of uniforms for girls.

Each set of uniform for boys requires 4m of material and each set of uniform for girls requires 3m of material the tailor has at most 600m of material to use.

The tailor makes, a profit of shs 8000 on each set of boy's uniform and shs 6000 on each set of girl uniform. The tailor is not sure of the number of sets of uniforms to make in order to maximize profit

Summary

Gender	Sets	cost	profits
Boys	At least 80	4m	8000
Girls	Not more than 100	3m	6000
Total		At most 600m	

Suppose x is number of boy's uniforms

And y is number of girl's uniforms

The tailor makes at least 80 sets of uniforms and not more than 100 sets of uniforms for girls.

$X \geq 80$, "at least" meaning the smallest should be 80

$Y \leq 100$. not more than "meaning the largest should be 100

Each set of uniform for boys requires 4m of material and each set of uniform for girls requires 3m of material the tailor has at most 600m of material to use.

$4X + 3Y \leq 600$, at most" meaning

$X \geq 0, Y \geq 0$, The non-negativity inequality

Objective function

The tailor makes, a profit of shs 8000 on each set of boy's uniform and shs 6000 on each set of girl uniform.

$$8000X + 6000Y$$

b) $4X + 3Y = 600$

X	0	150
Y	200	0

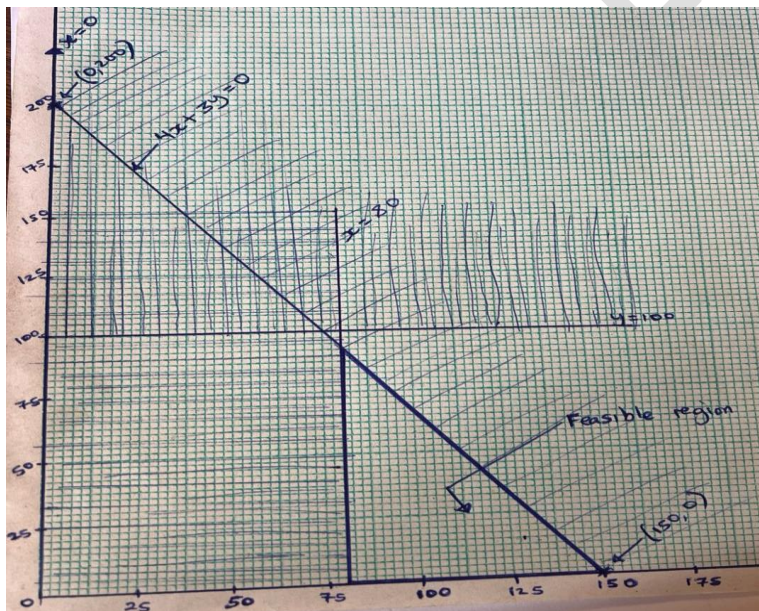
Points on the line (0, 200), (150, 0)

Using point (0,0), to test for unwanted region

$$4X + 3Y \leq 600$$

$$4(0) + 3(0) \leq 600$$

$0 \leq 600$ this is true, therefore the region where 0 is left unshaded



c). the apex of the triangle is (80, 90) giving 80 sets for boys and 90 sets for girls

the maximum amount of money is

$$= 8000X + 6000Y$$

$$= 8000(80) + 6000(90)$$

$$160,000 + 540,000$$

Shs 700,000

Section B

ITEM 3

James has been sent to a certain region to find out whether it is necessary to set up a new office for processing national identity cards, driving permits and passports, if the probability of a person having only one document is less than 50%.

James randomly interviewed 75 people in that region. He found that 18 people did not have any of the three documents, 30 people had driving permits and 20 people had passports. The number of people who had both national identity cards and driving permits only, was twice those who had passports and driving permits but not national identity cards. 4 people had passport only.

3 people had all the three documents. And 12 had driving permits only. James has to report about the number of people who have national identity cards but he has some missing information

- Help James to find the total number of people with national cards
- Determine if there is need for a new office in the region, give a reason for your answer

Summary

James randomly interviewed 75 people in that region.

$$n(\epsilon) = 75$$

He found that 18 people did not have any of the three documents,

$$n(NnDnP)' = 18$$

30 people had driving permits

$$n(D) = 30$$

20 people had passports.

$$n(P) = 20$$

The number of people who had both national identity cards and driving permits only, was twice those who had passports and driving permits but not national identity cards.

$$N(NnD) = 2 n(PnDnI')$$

4 people had passport only.

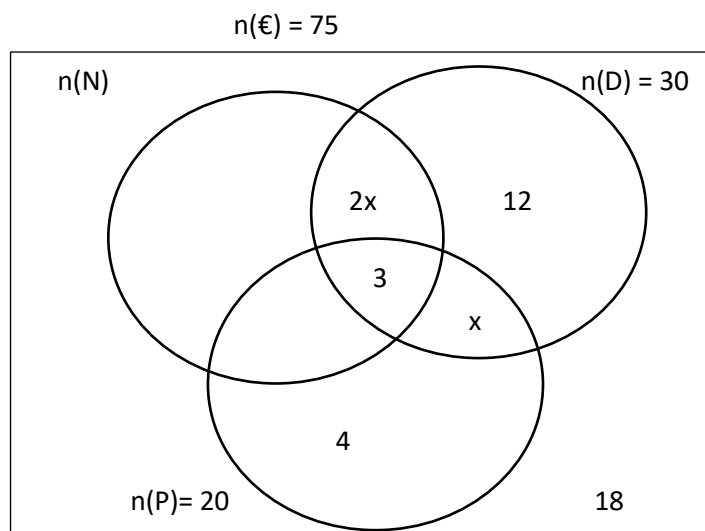
$$n(N'nD'nP) = 4$$

3 people had all the three documents.

$$n(NnDnP) = 3$$

And 12 had driving permits only

$$n(N \setminus nDnP) = 12$$



a) Let the total number of people with national cards = N

And those who had passports and driving permits be x

$$30 = 2x + 12 + x + 3$$

$$15 = 3x$$

$$5 = x$$

Taking the whole set gives

$$N + 12 + x + 4 + 18 = 75$$

$$N = 41$$

b) James has been sent to a certain region to find out whether it is necessary to set up a new office for processing national identity cards, driving permits and passports, if the probability of a person having only one document is less than 50%.

A new office should be built only if the probability of a person having only one document is less than 50%.

$$\text{the probability of a person having only one document} = \frac{\text{people having only one document}}{\text{total number of people}} = \frac{3}{75}$$

And $\frac{3}{75} \times 100$ which 4% is less than 50% therefore office must be built

Item 4

A farmer's cooperative society gave out fertilizers to a sample of maize farmers in the first season. The number of bags of maize harvested in the first season are given below

20 40 60 55 36 69 59 78 47 66

59 70 53 24 63 50 46 38 68 57
 30 65 58 61 57 86 77 54 29 88
 62 44 89 45 87 65 47 49 52 69
 41 80 37 56 74 27 76 58 79 39

The society wants

- (i) The data to be personal in a frequency distribution table with classes of interval 10 for easy interpretation
- (ii) To determine the number of bags of maize above which half of the farmers harvested so as to establish the real effect of the fertilizers
- (iii) To determine the number of farmers whose harvest is 44.5 bags of maize and below for re training before the second season but they lack the information

Task

- (a) Represent the data in a frequency distribution table
- (b) Determine the number of bags of maize above which half of the farmers harvested
- (c) Determine the number of farmers for re-training

Solution (a)

Class intervals	Class boundary	tallies	f	cf	Mid point (x)	F x
20- 29	19.5- 29.5	////	4	4	24.5	98
30- 39	29.5- 39.5	###	5	9	34.5	172.5
40 -49	39.5 -49.5	####	8	17	44.5	356
50 - 59	49.5 – 59.5	#####	12	29	54.5	654
60 - 69	59.5 – 69.5	#####	10	39	64.5	645
70 - 79	69.5 – 79.5	###	6	45	74.5	447
80 - 89	79.5 – 89.5	###	5	50	84.5	422.5
Total			50			

- b) Determine the number of bags of maize above which half of the farmers harvested

this is the median

$$m = l + \left(\frac{\frac{ef}{2} - cf}{f} \right) h \text{ where}$$

l is the lower limit of the median class

$\frac{ef}{2}$ is the number of observations

cf is the cumulative frequency of class preceding the median class

f is the frequency of the median class

h is the class size

Median class = $\left(\frac{\Sigma f + 1}{2}\right)$ th term which is 50 – 59 class

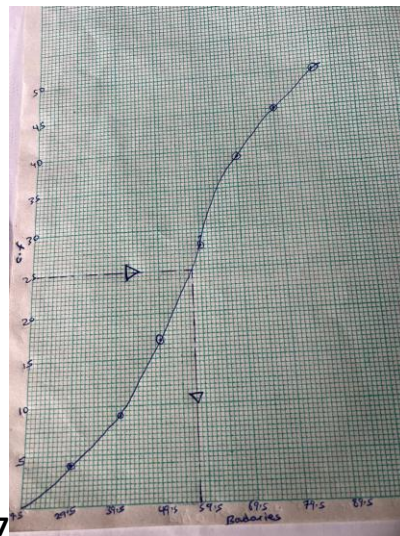
Now for $m = l + \left(\frac{\frac{\Sigma f}{2} - cf}{f}\right)h$

$$m = 50 + \left(\frac{\frac{50}{2} - 17}{12}\right) 10 = 56.6666 = 57$$

approximately half of the farmers harvested more than 57 bags of maize

Method 2

$$\frac{50}{100} \times 50 = 25\text{th position of the cumulative frequency}$$



From the graph = 57

- c) Determine the number of farmers for re-training

To determine the number of farmers whose harvest is 44.5 bags of maize and below for re-training before the second season

These include 20 – 44.5

These are (20 – 29) plus (30 – 39) plus (40 – 44.5)

$$4 + 5 + 3 = 12$$

12 farmers will have to be retrained for harvesting less than 44.5 bags of maize in the first season

Part 11

Item 5

Your sister bakes cakes and as received an order for a cake from your friend, Joseph, who needs a cylindrical cake with a base area of 6600.5 cm^2

Your sister suggested two payment plans to Joseph. He can pay shs 400,000 cash or pay a deposit of shs 200,000 and 2 equal installments of shs 110,000 each. However, Joseph has found it hard to decide on the payment plan.

Your sister has a triangular board that she wants to use to cut the circular tray on which the base of the cake will sit. Two sides of the triangular board measure 60cm and 48 cm , and the angle between these sides is 60°

Your sister is not sure whether the largest circular tray (inscribed circle) she can cut out of the triangular board will be enough for the base of the cake

Task

- A) Which of the two payment plans will you recommend to Joseph? Give a reason for your answer
- B) Determine whether the largest circular tray she plans to cut out of the triangular board will be enough for the base of the cake

Solution 5

- a) Cash

Advantages of Cash Payment

1. **Immediate payment completion** – Cash lets you finish paying right away without waiting. It's quick because you hand over the money and the purchase is done.
2. **No need for bank account records** – You don't have to have a bank account to use cash. Anyone can pay with it, even if they don't use a bank.
3. **Easy to use and understand** – Cash is simple. Most people know how to use it, and there's no need to learn about hire purchase and interest
4. **No transaction fees** – When you pay with cash, you don't have to pay extra fees. What you see is what you pay, with no surprises.
5. **Anonymity in payments** – Using cash can keep your purchases private. No one needs to know what you bought or where you spent your money.
6. **Cash releases you from pressure of thinking about the loan and endless calls from the seller in case another payment is due**
7. Cash keeps you away from **the tedious work** of making so many transactions
8. **Paying cash (400,000) is cheaper than installments(420,000)**
9. so many transactions may incur Joseph **an extra fee ie** transaction fees or transport fees every time he has to make an appointment

Or (other possible answer)

Paying in installments

Advantages of Paying in installments

The low deposit value makes it easier for the buyer to purchase and gives him time to look for the remainder

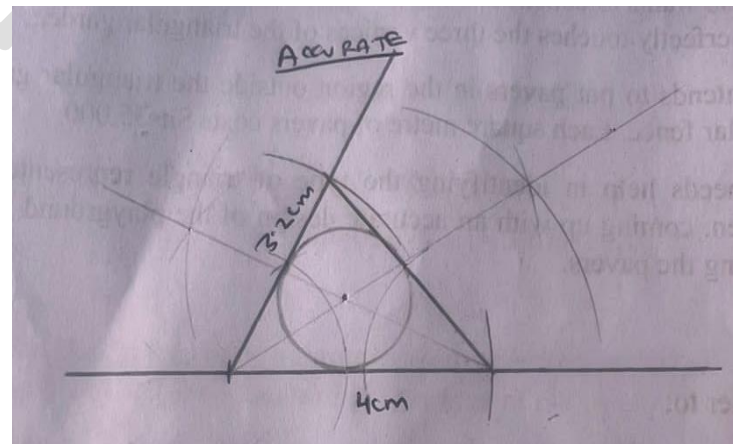
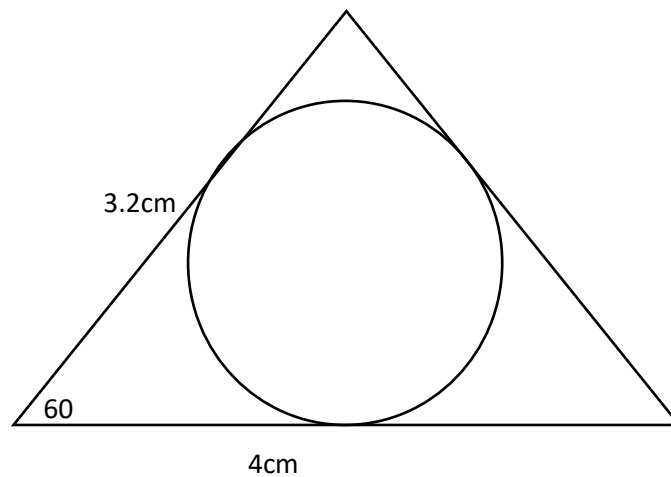
B) Determine whether the largest circular tray she plans to cut out of the triangular board will be enough for the base of the cake

Using scale

1cm to represent 15 actual cm

60cm will be 4cm and 48cm will be 3.2cm

Sketch



The radius from the accurate diagram is 1cm which is 15cm actual length

$$\text{Giving an area } \pi r^2 = \frac{22}{7} \times 15^2 = 707.14 \text{ cm}^2$$

The triangular board will be enough for the cake of base area (660.5cm²) since its largest circle is of area 707.14cm²

Item 6

Your brother wants to design a children playground. The playground will have a triangular garden and a circular fence around the garden. The two sides of the triangular garden will measure 50m and 70m, and the angle between them will be 45°

Your brother also wants to construct a circular fence around the garden such that the circular fence touches the vertices of the triangular garden

Your brother intends to put pavers in the region outside the triangular garden but inside the circular fence. Each square meter of pavers costs 35,000

Your brother needs help in identifying the type of triangle represented by the triangular garden, coming up with an accurate design of the playground as well as the cost of buying the pavers

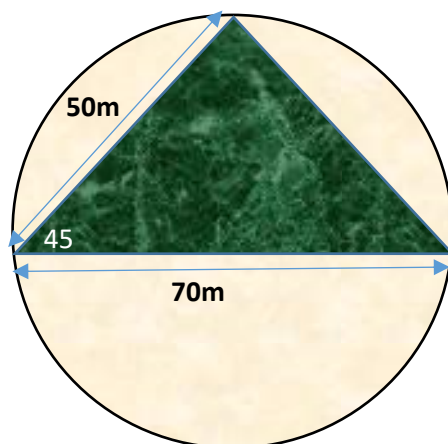
Task

Help your brother

- construct an accurate design of the children's play ground
- identify the type of triangle represented by the triangular garden and give a reason for your answer
- determine the amount of money needed to buy the pavers

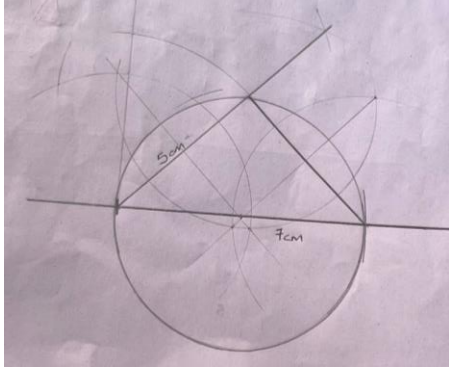
solution

sketch



scale each cm on the diagram below will represent 10m

Accurate



b) method 1

by measuring the missing side of the triangle using a ruler we find out its 5cm (50m) which makes the triangle isosceles but if 2 sides are equal therefore two angles are equal, if the triangle has two 45 angles the rest is 90. Therefore it's a right angled isosceles triangle

method 2

by measuring the missing angles of the triangle using a protractor, we find out 2 angles are 45 and one 90 degrees meaning it is a right angled isosceles triangle

method 3

by using the cosine formula to find out the missing side

$$. a^2 = b^2 + c^2 + 2bc \cos \theta$$

$$. a^2 = 5^2 + 7^2 + 2 \times 5 \times 7 \cos 45$$

$$. a^2 = 25 + 49 + 70 \cos 45$$

$$. a^2 = 25$$

a = 5cm, therefore triangle is isosceles, which makes the triangle isosceles but if 2 sides are equal therefore two angles are equal, if the triangle has two 45 angles the rest is 90. Therefore it's a right angled isosceles triangle

C) area of the circle – area of the triangle, by measuring the radius is 3.5cm which is 35m and height is 5cm (50m)

$$. \pi r^2 - \frac{1}{2} \times b \times h$$

$$. \frac{22}{7} \times 35^2 - \frac{1}{2} \times 50 \times 50$$

3850- 500 = 3350 square meters

Multiplied by the cost of each paver gives $35,000 \times 3350 = \text{shs } 117,250,000$

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